

## Investigations on the Reduction of Package Densities in Coplanar Circuits

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**Abstract** — One of the advantages of coplanar circuit design over microstrip design is the possibility to achieve higher package densities. The paper presents a modified method to calculate generalized scattering parameters for neighboured coplanar lines. A full-wave method is used to analyze important parameters as insertion loss, isolation and reflection not only for the coplanar mode but also for the conversion to the coupled slot line mode. Results for various structures on a GaAs-substrate are presented.

### INTRODUCCIÓN

During the last few years coplanar waveguide (CPW) technique in state-of-the-art MMIC's became widely used. The coplanar circuits offer a couple of significant advantages over other designs mostly done in microstrip technique. From the technological point of view realization of the circuit on one side of the substrate is important. No via holes are needed and lumped elements as well as active parts can be integrated simply. Whereas from the designers point of view variation of the slot width and the width of the center conductor gives two degrees of freedom in determining line parameters, for example the characteristic impedance, once the substrate height is chosen. Most parts of the electromagnetic fields from the coplanar mode are concentrated in the slot area. Therefore higher package densities compared with microstrip design should become possible. On the other hand coplanar lines are guiding two fundamental modes. First the technical relevant coplanar (even) mode and second the parasitic coupled slotline (odd) mode. The latter one is excited in electrically or geometrically nonsymmetrical discontinuities; the mode shows a dispersive and radiating behaviour. In order to achieve higher package densities the knowledge of parameters as transmission, isolation and coupling coefficient between neighboured coplanar lines are very important. Not even for the coplanar mode but also for the unwanted conversion to the coupled slotline mode.

In the next section of this paper, the numerical method to derive the line parameters of the coupled coplanar line structure is described very briefly. A spectral domain method with subsectional basis functions as expansion functions is used to calculate the eigenvalues including losses due to nonideal dielectrics and metallization. As an extension to the definition of characteristic impedances in [1], an application to lossy lines is presented. Based on the four complex propagation constants and the four by four matrix of characteristic impedances, a modified algorithm to generate generalized scattering parameters for coupled coplanar lines of length  $L$  is introduced.

The last section presents results for transmission, isolation and coupling coefficients of parallel coplanar lines. Not even for the coplanar mode but also for the conversion to the coupled slotline mode. The method is employed to analyze various coplanar structures printed on a GaAs substrate of finite height  $410\mu\text{m}$  without lower groundmetallization and shielding. Numerical results as a function of frequency and distance for approximately  $50\Omega$  lines are demonstrated.

### THEORY

As usual in spectral domain theory, all planar metallizations are assumed infinitely thin conductors. Nonideal metallization can be modeled by plane impedance surfaces. Considering of the coplanar structure as an aperture in a planar conducting screen of infinite extent separating two half spaces of different electromagnetic properties splits the original problem into two simpler ones [2]. After introducing magnetic surface currents  $\mathbf{M}_s$  to restore the electric slot fields  $\mathbf{e}_z \times \mathbf{E}_s = -\mathbf{M}_s$ , the apertures are replaced by planar metallization (Fig. 1). The magnetic currents, related as shown in the figure, radiate an electromagnetic field either in the upper region (a) or the lower region (b). The remaining boundary condition to be applied is the continuity of the tangential magnetic field on the slot apertures  $x \in \Omega_{slot}$ .

$$\mathbf{e}_z \times [\mathbf{H}^\dagger(x, z=0) - \mathbf{H}^\dagger(x, z=0)] = \mathbf{0}. \quad (1)$$

Each field component involved in (1) can be expressed by an integral form

$$\begin{pmatrix} \mathbf{H}^\dagger \\ \mathbf{H}^\perp \end{pmatrix} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{\mathbf{G}}_{H,M} \cdot \begin{pmatrix} \tilde{\mathbf{M}}^\dagger \\ \tilde{\mathbf{M}}^\perp \end{pmatrix} e^{jk_x x} dk_x \quad (2)$$

where  $\tilde{\mathbf{G}}_{H,M}$  is the dyadic Green's function calculated in an iterative manner for a multilayered dielectric substrate backed by a plane impedance surface. The tilde is indicating the spectral domain. Combination of (1) and (2) leads to an integral equation which can be solved by the method of moments using rectangular and triangular shaped subsectional basis functions. Then Galerkin's method is applied to project the integral equation onto a set of linear equations  $\mathbf{Z} \cdot \mathbf{M}_s = \mathbf{0}$ . Afterwards, the complex propagation constant  $\gamma_\nu$  of mode  $\nu$  can be determined by searching for  $\det\{\mathbf{Z}(\gamma_\nu)\} = 0$ . After solving this eigenvalue problem, calculation of the modal power  $P^\nu$ , transported by each mode, is straightforward.

$$P^\nu = \frac{1}{2} \iint_{A_{cross}} [\mathbf{E}_{tr}^\nu \times (\mathbf{H}_{tr}^\nu)^*] \cdot \mathbf{i}_y dA. \quad (3)$$

Following [3, 1] in assuming orthogonality of eigenvoltage and eigencurrent vectors of different modes (3) is used to define

$$\text{diag}\{P^1, \dots, P^*, \dots, P^{N-1}\} = \frac{1}{2} \mathbf{V} \mathbf{I}^{*T} \quad (4)$$

where  $\mathbf{V}$  and  $\mathbf{I}$  denote the matrices containing the mode quantities  $\mathbf{V}^\nu$  and  $\mathbf{I}^\nu$  for all modes  $\nu = 1 \dots (N-1)$  of a multistrip structure containing  $N$  conductors. Mode voltages can be derived from the magnetic currents in the slots by integration. According to this definition characteristic impedances have to be defined by

$$Z_{m,n}^L = \frac{V_{m,n}}{I_{m,n}}. \quad (5)$$

Starting with the complex propagation constants, the characteristic impedances and eigencurrent vectors of each mode, in [4, 5] algorithms to calculate scattering parameters of coupled microstrip lines are reported. The algorithm described there is modified here to look for the generalized scattering parameters of parallel coplanar lines with cross section illustrated in Fig. 2. After defining one of the five conductors as common earth, four line currents and line voltages at the beginning (subscript 0) and at the end (subscript L) of the coupled section can be introduced (Fig. 3). Electromagnetic fields within the structure can be obtained as a superposition of the four fundamental modes existing. Line currents may be constructed from the eigencurrents by

$$\begin{pmatrix} \mathbf{I}_0^{TL} \\ \mathbf{I}_L^{TL} \end{pmatrix} = \begin{pmatrix} \mathbf{M}_I & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_I \end{pmatrix} \cdot \begin{pmatrix} \mathbf{I}_0 \\ \mathbf{I}_L \end{pmatrix} \quad (6)$$

where

$$\mathbf{M}_I = ((\mathbf{I}^1)^T, \dots, (\mathbf{I}^4)^T) \cdot \text{diag}\left(\frac{1}{I_1^1}, \dots, \frac{1}{I_1^4}\right). \quad (7)$$

Assuming a TEM-line approximation, transmission line theory

$$V_{i,L}^\nu(L) = V_{i,0}^\nu \cosh(\gamma_\nu L) - Z_{L,i}^\nu I_{i,0}^\nu \sinh(\gamma_\nu L), \quad (8)$$

$$I_{i,L}^\nu(L) = I_{i,0}^\nu \cosh(\gamma_\nu L) - \frac{V_{i,0}^\nu}{Z_{L,i}^\nu} \sinh(\gamma_\nu L) \quad (9)$$

can be applied to connect currents and voltages of each mode  $\nu$  at the ends of every line  $i$ . Excitation of the fundamental modes and analysis of the resulting total voltages by superimposing, results in

$$\left. \begin{aligned} \mathbf{V} &= \mathbf{Z}_L \cdot \mathbf{I} \\ \mathbf{V}_i^{TL} &= \sum_{\nu=1}^4 V_i^\nu \end{aligned} \right\} \Rightarrow \mathbf{V}^{TL} = \mathbf{M}_V \cdot \mathbf{I} \quad (10)$$

where

$$\mathbf{V}^{TL} = \begin{pmatrix} \mathbf{V}_0^{TL} \\ \mathbf{V}_L^{TL} \end{pmatrix}, \quad \mathbf{V} = \begin{pmatrix} \mathbf{V}_0 \\ \mathbf{V}_L \end{pmatrix}. \quad (11)$$

Insertion of (6) in (10) gives

$$\mathbf{V}^{TL} = \mathbf{M}_V \cdot \begin{pmatrix} \mathbf{M}_I & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_I \end{pmatrix}^{-1} \cdot \mathbf{I}^{TL} = \mathbf{Z} \cdot \mathbf{I}^{TL} \quad (12)$$

where  $\mathbf{Z}$  is the impedance matrix of the 8-port coupled line structure. The scattering matrix could be determined from this  $\mathbf{Z}$ -matrix directly. Nevertheless, it is important to note, the resulting scattering matrix is not describing the behaviour of the neighbouring coplanar lines but that in a sense of coupled microstrip lines. In order to get the scattering parameters of parallel copla-

nar lines, a transformation of the line voltages and line currents into the even and odd voltages as well as currents of both coplanar lines  $I$  and  $II$  has to be made. The overall procedure is illustrated in Fig. 4. After introducing the matrices

$$\mathbf{M}_V^{cop} = \frac{1}{2} \begin{pmatrix} -1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \quad \mathbf{M}_I^{cop} = \frac{1}{2} \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}, \quad (13)$$

a transformation

$$\begin{pmatrix} \mathbf{X}_0^{cop} \\ \mathbf{X}_L^{cop} \end{pmatrix} = \begin{pmatrix} \mathbf{M}_X^{cop} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_X^{cop} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{X}_0^{TL} \\ \mathbf{X}_L^{TL} \end{pmatrix} \quad (14)$$

can be done where  $X = I, V$ . Reformulation of (12) with the help of (13) provides the "coplanar" impedance matrix  $\mathbf{Z}_{cop}$ . Next step is an element by element normalization  $Z_{m,n}^{norm} = Z_{m,n}^{cop} / \sqrt{Z_{L,m} Z_{L,n}}$  with the characteristic impedance of the feeding lines (here the characteristic impedance for the even- and odd-mode of the attached coplanar lines). Afterwards the  $S$ -matrix can be given in terms of the modified, normalized impedance matrix:

$$\mathbf{S} = (\mathbf{Z}_{norm} + \mathbf{I})^{-1} \cdot (\mathbf{Z}_{norm} - \mathbf{I}). \quad (15)$$

## RESULTS

In the numerical results shown here, the considered parallel lines are printed on a  $410\mu\text{m}$ -GaAs substrate ( $\epsilon_r = 12.9$ ) with a slot width of  $49\mu\text{m}$ . The width of the inner conductor is  $77\mu\text{m}$ . Characteristic impedance for the coplanar mode of such lines is approximately  $50\Omega$ .

One of our design rules for CPW circuits is given by  $a \geq w+2s$  (Fig. 2). In order to check this rule a choice of  $a = w+2s = 175\mu\text{m}$  has been made for the first calculations presented here. The results are shown in Fig. 5 and Fig. 6 for the frequency range of 0-40GHz. Fig. 5 depicts some of the even mode scattering parameters; with  $S_{11}^{ee}$  the reflection coefficient,  $S_{21}^{ee}$  the isolation, and  $S_{41}^{ee}$  the coupling from line  $I$  to line  $II$ . As can be seen from the figure,  $S_{11}^{ee}$  is better than -48dB as well as the coupling ( $|S_{41}^{ee}| \leq -47.6\text{dB}$ ) is. From the designers point of view even the isolation characteristics for the even mode with  $|S_{21}^{ee}| \leq -30\text{dB}$  are sufficient. Very similar results for the conversion from the technical relevant even mode to the parasitic odd mode are obtained (Fig. 6). Here the isolation from port 1 to port 2 including a mode conversion is still better than -20dB.

In order to look for a possible reduction of the package density inside CPW circuits the distance  $a$  between both coplanar lines has been varied from  $a = 420\mu\text{m}$  down to  $a = 77\mu\text{m}$ , whereas for the latter structure  $a = w$  (Fig. 2). To give an impression for the influence of the distance  $a$  on the variation of the scattering parameters two has been depicted in Fig. 7 and Fig. 8. As can be seen in Fig. 7, the coupling coefficient is still below -40dB for  $a = 77\mu\text{m}$  which is enough for any technical applications. The isolation  $|S_{21}^{ee}|$  rises up to -17dB and therefore  $a = 77\mu\text{m}$  may not be acceptable from a designers point of view. Nevertheless, it is evident that coplanar lines are very suitable to reduce the package densities within MMIC's. As mentioned in the introduction a further reduction is possible by scaling the slot width and the

width of the inner conductor. For example a  $50\Omega$  coplanar line on the substrate described above could be realized with geometry parameters of  $s = 10\mu\text{m}$  and  $w = 15\mu\text{m}$ .

### CONCLUSION

Generalized scattering parameters of parallel coplanar lines on a GaAs substrate are illustrated in this paper. The algorithm to analyze these structures is described briefly. The analysis gives a straightforward method to answer questions from CPW circuit designers about package densities achievable. Comparison with equivalent microstrip results will be prepared for the oral presentation.

### REFERENCES

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### FIGURES

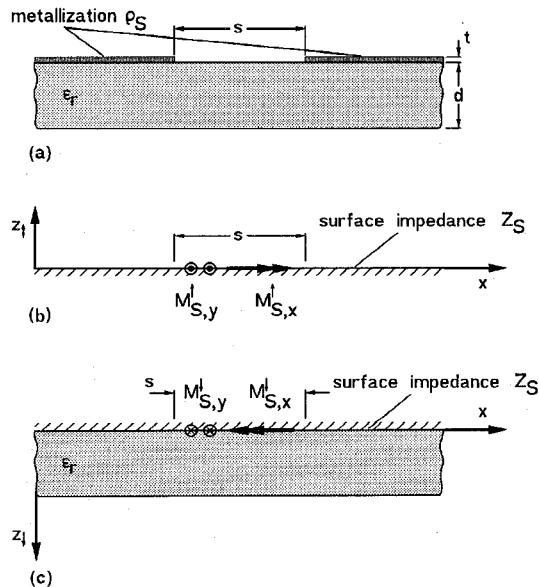


Fig 1. Step by step substitution of the original slot structure (a) by two simpler structures valid above (b) and below (c) the aperture.

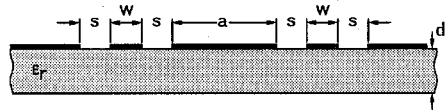


Fig 2. Coupled parallel coplanar waveguide.

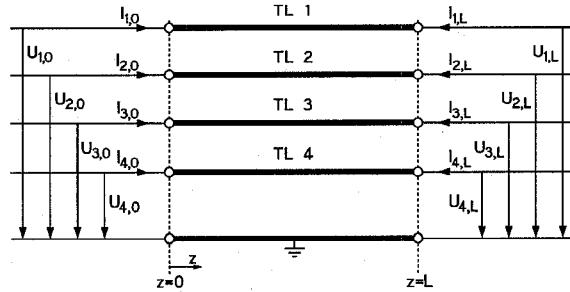


Fig 3. Schematic circuit for a coupled parallel coplanar section of length  $L$ .

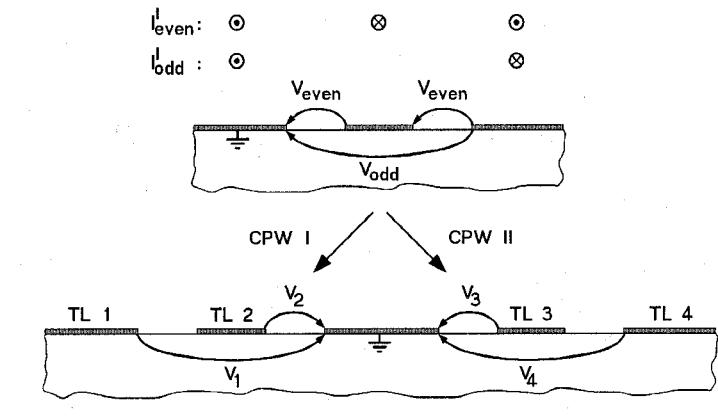


Fig 4. Schematic diagram describing the transformation from line voltages and line currents of the coupled line structure to those of the even and odd mode in the coplanar lines I and II.

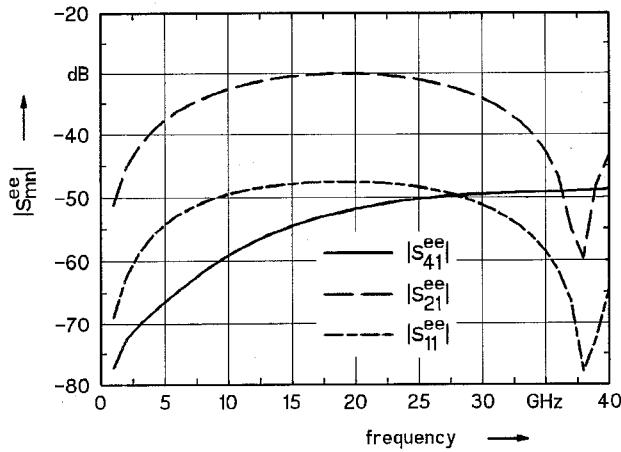


Fig 5. Numerical results for the even mode scattering parameters  $S_{11}^{ee}$  (reflection),  $S_{21}^{ee}$  (isolation) and  $S_{41}^{ee}$  (coupling);  $\epsilon_r = 12.9$ ,  $d = 410\mu\text{m}$ ,  $w = 77\mu\text{m}$ ,  $s = 49\mu\text{m}$ ,  $w = 49\mu\text{m}$ ,  $a = 175\mu\text{m}$  and  $L = 1.5\text{mm}$  (Fig. 2).

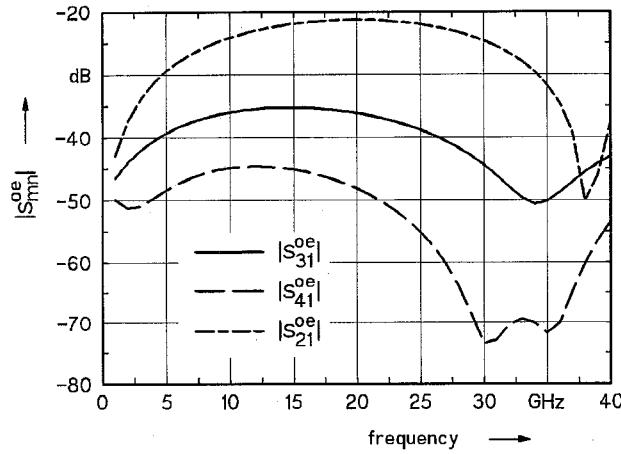


Fig 6. Numerical results for the conversion to the odd mode  $S_{21}^{oe}$  (isolation),  $S_{31}^{oe}$  (transmission) and  $S_{41}^{oe}$  (coupling); parameters from Fig. 5.

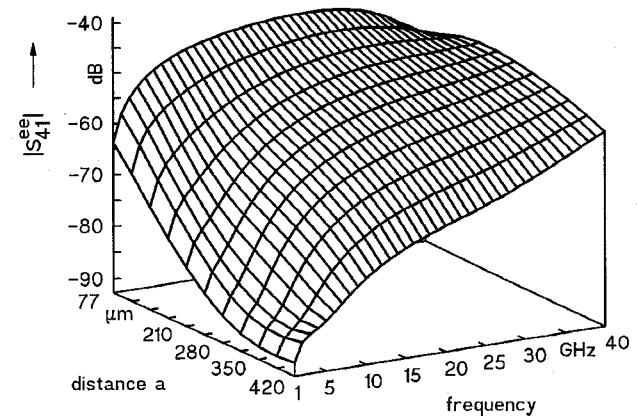


Fig 7. Numerical results for the coupling of the even mode versus frequency and distance  $a$ ; parameter from Fig. 5.

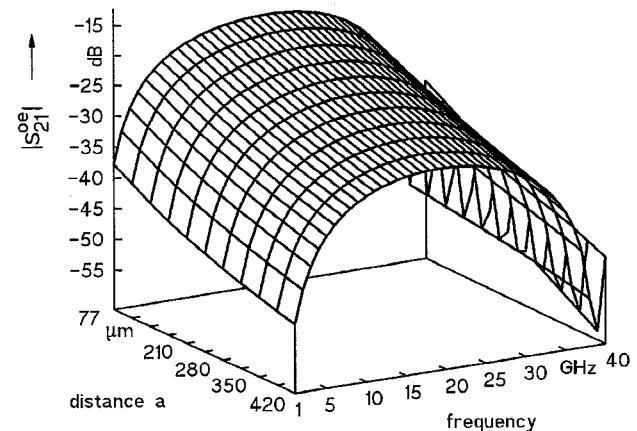


Fig 8. Numerical results for the isolation including a mode conversion from even to odd mode versus frequency and distance  $a$ ; parameter from Fig. 5.